Quantitative Spatial Models: Owens, Rossi-Hansberg, and Sarte (2020)

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Plan for Today

- Quantification of Owens, Rossi-Hansberg, and Sarte (2020)
- Model
- Equilibrium
- Estimation (assuming we have the data we need)
- Solution algorithm for counterfactuals

Model (1/3)

- Geography: $\{\bar{T}_j^b, \bar{T}_j^r, A_j, F_j, \sigma_j, \lambda_{ij}, \kappa_{ij}\}$
- Open city: freely move in and out of Detroit
- Atomistic local firms with CRS technology

$$Y_j = A_j \left(\frac{L_j}{T_j^b}\right)^{\alpha} L_j^{\beta} (T_j^b)^{1-\beta} \implies \frac{Y_j}{T_j^b} = \left(A_j \ell_j^{\alpha}\right) \ell_j^{\beta}$$

- Local labor demand

$$L_j = \left(\frac{A_j \beta}{w_j}\right)^{\frac{1}{1-\beta-\alpha}} T_j^b, \quad 1-\beta > \alpha \tag{1}$$

- Competitive commercial land market $\implies$ zero profits $\implies$ commercial rent

$$q_j^b = (1-\beta) A_j^{\frac{1}{1-\beta-\alpha}} \left(\frac{\beta}{w_j}\right)^{\frac{\beta+\alpha}{1-\beta-\alpha}}$$

Model (2/3)

- Indirect utility

$$U_{ij}(s) = \frac{sR_j^{\sigma_j}}{\kappa_{ij}} w_i \left(q_j^r\right)^{\gamma-1}, \quad \sigma_j > 1-\gamma$$

- Expected utility living in j

$$U_j = \Gamma\left(\frac{\theta-1}{\theta}\right) R_j^{\sigma} \left(q_j^r\right)^{\gamma-1} \left(\sum_{i=1}^{J} \lambda_{ij} \left(\frac{w_i}{\kappa_{ij}}\right)^{\theta}\right)^{\frac{1}{\theta}} \tag{2}$$

- Commuting flows

$$\pi_{ij} = \frac{\lambda_{ij} \left(w_i/\kappa_{ij}\right)^{\theta}}{\sum_{n=1}^{J} \lambda_{nj} \left(w_n/\kappa_{nj}\right)^{\theta}} \tag{3}$$

- Residential demand

$$H_j = \frac{1-\gamma}{q_j^r} \sum_{i=1}^{J} \pi_{ij} w_i, \quad R_j H_j = T_j^r \implies q_j^r = \frac{(1-\gamma)R_j}{T_j^r} \sum_{i=1}^{J} \pi_{ij} w_i \tag{4}$$

Model (3/3)

- Atomistic residential developers

$$\max_{h_j} \; q_j^r h_j - V h_j^{\nu} - F_j \implies h_j = \left(\frac{q_j^r}{\nu V} \right)^{\frac{1}{\nu - 1}}$$

- Free entry (subject to residential zoning)

$$q_j^r h_j \geq V h_j^{\nu} + F_j \implies R_j \geq \frac{\nu V \left(\frac{F_j}{(\nu-1)V} \right)^{\frac{\nu-1}{\nu}}}{(1-\gamma) \sum_{i=1}^{J} \pi_{ij} w_i} T_j^r \tag{5}$$

- Mass of developers (conditional positive entry)

$$n_j h_j = R_j H_j = T_j^r \leq \bar{T}_j^r \implies n_j = \frac{(T_j^r)^{\frac{\nu}{\nu-1}}}{\left(\frac{1-\gamma}{\nu V} R_j \sum_{i=1}^{J} \pi_{ij} w_i \right)^{\frac{1}{\nu-1}}}$$

Open City Equilibrium (1/2)

- Exogenous parameters: $\{\alpha, \beta, \gamma, \theta, \nu, V, \bar{u}, T_j^b, \bar{T}_j^r, A_j, F_j, \sigma_j, \lambda_{ij}, \kappa_{ij}\}$
- Endogenous variables: $\{R_j, T_j^r, q_j^r, w_j, \pi_{ij}\}$ and $\{\Omega^F, \Omega^S\}$

1. Free mobility: (2) + (4)

$$R_j = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma} \left\{ \sum_{i=1}^{J} \pi_{ij} w_i \right\}^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right) \left(T_j^r\right)^{1-\gamma} \left[\sum_{i=1}^{J} \lambda_{ij} \left(\frac{w_i}{\kappa_{ij}}\right)^{\theta} \right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j+\gamma-1}}, \quad \forall j \in \Omega^F \cup \Omega^S$$

2. Developer entry: (5)

$$R_j \geq \frac{\nu V \left(\frac{F_j}{(\nu-1)V}\right)^{\frac{\nu-1}{\nu}}}{(1-\gamma) \sum_{i=1}^{J} \pi_{ij} w_i} T_j^r, \quad \forall j \in \Omega^F \cup \Omega^S$$

Open City Equilibrium (2/2)

3. Residential rent: (4)

$$q_j^r = (1-\gamma)\frac{R_j}{T_j^r}\sum_{i=1}^{J}\pi_{ij}w_i, \quad \forall j \in \Omega^F \cup \Omega^S$$

4. Labor market clearing: (1) + labor supply

$$\left(\frac{A_i\beta}{w_i}\right)^{\frac{1}{1-\beta-\alpha}} T_i^b = \sum_{j=1}^{J}\pi_{ij}R_j, \quad \forall i \in \Omega$$

5. Commuting flows: (3)

$$\pi_{ij} = \frac{\lambda_{ij}\left(w_i/\kappa_{ij}\right)^{\theta}}{\sum_{n=1}^{J}\lambda_{nj}\left(w_n/\kappa_{nj}\right)^{\theta}}, \quad \forall i,j \in \Omega$$

- Treatment of zeros: For vacant neighborhoods ($j \in \Omega^V$), $R_j = T_j^r = q_j^r = \pi_{ij} = 0\ \forall i$

Estimation Overview

- Data: w_j, q_j^r, R_j, L_j, n_j, T_j^b, T_j^r, π_{ij}, κ_{ij}, Ω^F, Ω^S, Ω^V
- Three sets of parameters to be estimated

1. Citywide parameters: $\{\alpha, \beta, \gamma, \theta, \nu, V, \bar{u}\}$
2. Pair specific parameters: $\{\lambda_{ij}\}$
3. Neighborhood specific parameters: $\{\bar{T}_j^r, A_j, F_j, \sigma_j\}$

- Model inversion: parameters are chosen such that the model exactly rationalizes the data

Estimate Citywide and Pair Specific Parameters

- Calibrate $\alpha = 0.06$, $\beta = 0.80$, $\gamma = 0.76$, $\nu = 2.50$; normalize $\bar{u} = 1$
- Estimate V (scale of residential construction cost) to match mean number of active contractors n_j

$$\frac{1}{J}\sum_{j=1}^{J} n_j = \frac{1}{J}\sum_{j=1}^{J} (\nu V)^{\frac{1}{\nu-1}} \left(q_j^r\right)^{\frac{\nu}{\nu-1}} (1-\gamma) R_j \sum_{i=1}^{J} \pi_{ij} w_i \implies V$$

- Estimate θ (commuting elasticity) from the gravity equation (3)

$$\log \pi_{ij} = -\theta \log \kappa_{ij} + \mu_i + \mu_j + \varepsilon_{ij} \implies \theta$$

- Estimate λ_{ij} (scale of Fréchet distribution) from

$$\frac{\pi_{ij}}{\pi_{jj}} = \frac{\lambda_{ij}}{\lambda_{jj}} \left(\frac{w_i}{w_j}\right)^{\theta} \left(\frac{\kappa_{ij}}{\kappa_{jj}}\right)^{-\theta} \implies \lambda_{ij} = \pi_{ij} \left(\frac{\kappa_{ij}}{w_i}\right)^{\theta} \text{ normalizing } \left(\frac{w_j}{\kappa_{jj}}\right)^{\theta} \frac{\lambda_{jj}}{\pi_{jj}} = 1$$

Estimate Neighborhood Specific Parameters (1/2)

- F_j and $\bar{T}_j^r$ cannot be simultaneously inferred from the data for the same neighborhood

- To estimate F_j (fixed costs)

$$F_j = \begin{cases} (\nu - 1)V \left(\frac{q_j^r}{\nu F}\right)^{\frac{\nu}{\nu-1}}, & \text{if } j \in \Omega^S \\ \text{median}_{j' \in \Omega^S} \; F_{j'}, & \text{if } j \in \Omega^F \\ F_{j'} \text{ in the nearest partially developed neighborhood}, & \text{if } j \in \Omega^V \end{cases}$$

- To estimate $\bar{T}_j^r$ (upper bound for residential land)

$$\bar{T}_j^r = \begin{cases} \text{data counterpart}, & \text{if } j \in \Omega^S \\ (1-\gamma)\frac{R_j}{q_j^r}\sum_{i=1}^{J}\pi_{ij}w_i, & \text{if } j \in \Omega^F \\ \text{data counterpart}, & \text{if } j \in \Omega^V \end{cases}$$

Estimate Neighborhood Specific Parameters (2/2)

- To estimate A_i (productivity)

$$A_i = \frac{w_i}{\beta} \left(\frac{1}{T_i^b} \sum_{j=1}^{J} \pi_{ij} R_j \right)^{1-\beta-\alpha} \qquad \forall i \in \Omega$$

- To estimate σ_j (residential externalities)

$$\sigma_j = \begin{cases} (\log R_j)^{-1} \log \left(\dfrac{\bar{u}(1-\gamma)^{1-\gamma} \left\{ \sum_{i=1}^{J} \pi_{ij} w_i \right\}^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right) \left(T_j^r\right)^{1-\gamma} \left[\sum_{i=1}^{J} \lambda_{ij} \left(\frac{w_i}{\kappa_{ij}}\right)^{\theta} \right]^{\frac{1}{\theta}}} \right) - \gamma + 1, & j \in \Omega^F \cup \Omega^S \\ \sigma_{j'} \text{ in the nearest partially developed neighborhood}, & j \in \Omega^V \end{cases}$$

- Done!

Solution Algorithm for Counterfactuals

- We want to solve for counterfactual equilibrium
- Change a subset of exogenous parameters (holding others fixed), solve for endogenous variables and compare them with the baseline equilibrium
- Policy: coordinating vacant neighborhoods ($\Omega^V \to \Omega^S$ or Ω^F)
- Implementation: fixed-point algorithm

Fixed-Point Algorithm (1/3)

1. Guess a vector of wages, $\mathbf{w}^0 = [w_1^0, ..., w_J^0]$

2. Calculate commuting flows, $\pi_{ij}(\mathbf{w}^0)$

$$\pi_{ij}(\mathbf{w}^0) = \frac{\lambda_{ij}\left(\frac{w_i^0}{\kappa_{ij}}\right)^{\theta}}{\sum_{n=1}^{J}\lambda_{nj}\left(\frac{w_n^0}{\kappa_{nj}}\right)^{\theta}}, \quad \forall i, j \in \Omega$$

3. Calculate residential land, $T_j^r(\mathbf{w}^0)$, assuming no upper bound $\bar{T}_j^r$

$$T_j^r(\mathbf{w}^0) = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma}\left[\left(\frac{(v-1)V}{F_j}\right)^{\frac{v-1}{v}}\left(\frac{1-\gamma}{vV}\right)\right]^{\sigma_j+\gamma-1}}{\Gamma\left(\frac{\theta-1}{\theta}\right)\left\{\sum_{i=1}^{I}\lambda_{ij}\left[w_i^0/\kappa_{ij}\right]^{\theta}\right\}^{\frac{1}{\theta}}}\right)^{\frac{1}{\sigma_j}}\left(\sum_{i=1}^{J}\pi_{ij}(\mathbf{w}^0)w_i^0\right)$$

Fixed-Point Algorithm (2/3)

4. Obtain actual residential land, recognizing the fact that there is an upper bound

$$T_j^r(\mathbf{w}^0) = \min\{T_j^r(\mathbf{w}^0), \bar{T}_j^r(\mathbf{w}^0)\}$$

5. Calculate residential population, $R_j(\mathbf{w}^0)$

$$R_j(\mathbf{w}^0) = \left(\frac{\bar{u}(1-\gamma)^{1-\gamma} \left\{ \sum_{i=1}^J \pi_{ij}(\mathbf{w}^0) w_i^0 \right\}^{1-\gamma}}{\Gamma\left(\frac{\theta-1}{\theta}\right) \left(T_j^r(\mathbf{w}^0)\right)^{1-\gamma} \left[\sum_{i=1}^J \lambda_{ij} \left(\frac{w_i^0}{\kappa_{ij}}\right)^{\theta}\right]^{\frac{1}{\theta}}} \right)^{\frac{1}{\sigma_j+\gamma-1}}$$

6. Calculate labor demand, labor supply, and excess labor demand

$$L_i^D(\mathbf{w}^0) = \left(\frac{A_i \beta}{w_i^0}\right)^{\frac{1}{1-\beta-\alpha}} T_i^b, \quad L_i^S(\mathbf{w}^0) = \sum_{j=1}^J \pi_{ij}(\mathbf{w}^0) R_j(\mathbf{w}^0), \quad X_i(\mathbf{w}^0) = L_i^D(\mathbf{w}^0) - L_i^S(\mathbf{w}^0)$$

Fixed-Point Algorithm (3/3)

7. If $\sum_{i=1}^{J} |X_i(\mathbf{w}^0)| \geq \epsilon$, update the vector of wages

$$\mathbf{w}^1 = \mathbf{w}^0 + \delta \mathbf{X}(\mathbf{w}^0)$$

   where δ is the adjustment factor (not too large to ensure convergence)

8. Use $\mathbf{w}^1$ as the initial guess and iterate steps 1-7 until convergence

- Obtain $\{R_j, T_j^r, q_j^r, w_j, \pi_{ij}\}$ in the counterfactual equilibrium
- See Section 4 in Online Appendix B for details