Dynamic Quantitative Spatial Models: Desmet, Nagy, and Rossi-Hansberg (2018) Cruz and Rossi-Hansberg (2023)

Xianglong (Sean) Kong

February 20, 2024

ECON 33550: Spatial Economics

Desmet, Nagy, and Rossi-Hansberg (2018)

Overview

- Endogenous growth through innovation and diffusion
- Sources of dynamics: migration and innovation
- Tractability: agents in the model make static migration and innovation decisions
  - In contrast to CDP and KLR, where agents still make dynamic decisions (solve DP problem)
- Possible to recover fundamentals through model inversion

Equilibrium (1/3)

- Parameters: $\{\beta, \rho, \lambda, \Omega, \alpha, \theta, \mu, \gamma_1, \gamma_2, \xi, \nu, \eta\}$
- Trade and migration costs: $\{\varsigma, m\}$
- Initial condition: $\{\tau_0, \bar{a}, \bar{L}_0, H\}$
- Endogenous variables: $\{u_t, \bar{L}_t, \phi_t, R_t, w_t, P_t, \tau_t, T_t\}$

1. Firm optimization:

$$\mu p_t^\omega(r,r)\phi_t^\omega(r) z_t^\omega(r) L_t^\omega(r)^{\mu-1} = w_t(r)$$
$$\gamma_1 p_t^\omega(r,r)\phi_t^\omega(r)^{\gamma_1-1} z_t^\omega(r) L_t^\omega(r)^{\mu} = \xi w_t(r)\nu\phi_t^\omega(r)^{\xi-1}$$

2. Labor market clearing:

$$\bar{L}_t^\omega(r) = \frac{L_t^\omega(r)}{\mu}\left[\mu + \frac{\gamma_1}{\xi}\right]$$

3. Marginal cost and trade share:

$$mc_t(r) \equiv \left[\frac{1}{\mu}\right]^{\mu}\left[\frac{\nu\xi}{\gamma_1}\right]^{1-\mu}\left[\frac{\gamma_1 R_t(r)}{w_t(r)\nu(\xi(1-\mu)-\gamma_1)}\right]^{(1-\mu)-(\gamma_1/\xi)} w_t(r)$$
$$\pi_t(s,r) = \frac{T_t(r)[mc_t(r)\varsigma(r,s)]^{-\theta}}{\int_S T_t(u)[mc_t(u)\varsigma(u,s)]^{-\theta}du}$$

Equilibrium (2/3)

4. Trade balance:

$$w_t(r)H(r)\bar{L}_t(r) = \int_S \pi_t(s,r)w_t(s)H(s)\bar{L}_t(s)ds$$

5. Land rent from Bertrand competition:

$$R_t(r) = \left[\frac{\xi - \mu\xi - \gamma_1}{\mu\xi + \gamma_1}\right] w_t(r)\bar{L}_t(r)$$

6. Migration flow:

$$H(r)\bar{L}_t(r) = \frac{u_t(r)^{1/\Omega} m_2(r)^{-1/\Omega}}{\int_S u_t(v)^{1/\Omega} m_2(v)^{-1/\Omega} dv}\bar{L}$$

7. Utility:

$$u_t(r) = \bar{a}(r)\bar{L}_t(r)^{-\lambda}\frac{\xi}{\mu\xi + \gamma_1}\frac{w_t(r)}{P_t(r)}$$

$$P_t(s) = \Gamma\left(\frac{-\rho}{(1-\rho)\theta} + 1\right)^{-(1-\rho)/\rho}\left\{\int_S T_t(u)[mc_t(u)\varsigma(s,u)]^{-\theta} du\right\}^{-1/\theta}$$

Equilibrium (3/3)

8. Land market clearing:

$$\int_S H(r)\bar{L}_t(r)dr = \bar{L}$$

9. Productivity process:

$$\tau_t(r) = \phi_{t-1}(r)^{\theta\gamma_1}\left[\int_S \eta\tau_{t-1}(s)ds\right]^{1-\gamma_2}\tau_{t-1}(r)^{\gamma_2}$$

- An equilibrium exists and is unique if

$$\underbrace{\frac{\alpha}{\theta}}_{\text{prod. ext.}} + \underbrace{\frac{\gamma_1}{\xi}}_{\text{returns to R\&D}} \leq \underbrace{\lambda}_{\text{amen. ext.}} + \underbrace{1-\mu}_{\text{fixed factor}} + \underbrace{\Omega}_{\text{var. of taste}}$$

- An BGP exists and is unique if

$$\frac{\alpha}{\theta} + \frac{\gamma_1}{\xi} + \underbrace{\frac{\gamma_1}{[1-\gamma_2]\xi}}_{\text{dyn. agglo.}} \leq \lambda + 1 - \mu + \Omega$$

Data and Estimation (1/2)

- Data: $a(r), y(r), L(r), H(r), w(r)$, transportation networks, subjective well-being
- Calibration:

$$\beta = 0.95, \quad \rho = 0.75, \quad \Omega = 0.5, \quad \alpha = 0.06, \quad \theta = 6.5, \quad \mu = 0.8, \quad \xi = 125$$

- Estimate λ from the relationship between amenities and population:

$$\log(a(r)) = E(\log(\bar{a}(r))) - \lambda \log \bar{L}(r) + \varepsilon_a(r)$$

  Use exogenous productivity as an instrument for population

- Estimate γ_1, γ_2 from BGP growth rate:

$$\Delta \log y_{t+1}(c) = \alpha_1 + \alpha_2 \log \sum_{S_c} L_c(s)^{\alpha_3}, \quad \alpha_2 = \frac{1-\gamma_2}{\theta}, \quad \alpha_3 = \frac{\theta\gamma_1}{[1-\gamma_2]\xi}$$

- Choose ν to generate 2% real GDP growth
- Obtain $\varsigma(s, r)$ based on Allen and Arkolakis (2014)

Data and Estimation (2/2)

- Run model inversion to back out the rest of the location fundamentals
- Solve for $\frac{\bar{a}(r)}{u_0(r)}$ and $\tau_0(r)$ from the following equations:

$$\left[\frac{\bar{a}(r)}{u_0(r)}\right]^{-\theta} = \kappa_1 \bar{w}^{-(1+2\theta)} w_0(r)^{\theta} \bar{L}_0(r)^{-\lambda\theta} \int_S w_0(s)^{1+\theta} \bar{L}_0(s)^{1-\lambda\theta} H(s) \varsigma(r,s)^{-\theta} \left[\frac{\bar{a}(s)}{u_0(s)}\right]^{\theta} ds$$

$$\tau_0(r) = \bar{w}^{-(1+2\theta)} \left[\frac{\bar{a}(r)}{u_0(r)}\right]^{\theta} H(r) w_0(r)^{1+2\theta} \bar{L}_0(r)^{1-\alpha-\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\right]\theta}$$

- Obtain a measure of $u_0(r)$ from subjective well-being data: $u_0(r) = e^{1.8\check{u}(c(r))} \implies \bar{a}(r)$
- Solve for $m_2(r)$ from equations (37) and (38) in the paper

Solution Algorithm (1/4)

- Simulate the (baseline or counterfactual) economy forward

1. Update productivity:

$$\tau_t(r) = \phi_{t-1}(r)^{\theta\gamma_1} \left[\int_S \eta \tau_{t-1}(s) ds \right]^{1-\gamma_2} \tau_{t-1}(r)^{\gamma_2}$$

2. Solve for utility level using Lemma 3 (see below)
3. Compute population:

$$\bar{L}_t(r) = \frac{u_t(r)^{1/\Omega} m_2(r)^{-1/\Omega}}{\int_S u_t(v)^{1/\Omega} m_2(v)^{-1/\Omega} dv} \frac{\bar{L}}{H(r)}$$

4. Compute innovation from FOCs of firms:

$$\phi_t(r) = \left(\bar{L}_t(r) \right)^{1/\xi} \left[\frac{\gamma_1}{\nu(\mu\xi + \gamma_1)} \right]^{1/\xi}$$

Solution Algorithm (2/4)

5. Compute wages:

$$w_t(r) = \bar{w}\left[\frac{\bar{a}(r)}{u_t(r)}\right]^{-\frac{\theta}{1+2\theta}} \tau_t(r)^{\frac{1}{1+2\theta}} H(r)^{-\frac{1}{1+2\theta}} \bar{L}_t(r)^{\frac{\alpha-1+\left[\lambda+\frac{\gamma}{\xi}-[1-\mu]\right]}{1+2\theta}}$$

6. Compute land rent:

$$R_t(r) = \left[\frac{\xi-\mu\xi-\gamma_1}{\mu\xi+\gamma_1}\right] w_t(r)\bar{L}_t(r)$$

7. Compute marginal cost and price index:

$$mc_t(r) = \left[\frac{1}{\mu}\right]^{\mu}\left[\frac{\nu\xi}{\gamma_1}\right]^{1-\mu}\left[\frac{\gamma_1 R_t(r)}{w_t(r)\nu(\xi(1-\mu)-\gamma_1)}\right]^{(1-\mu)-(\gamma_1/\xi)} w_t(r)$$

$$P_t(r) = \Gamma\left(\frac{-\rho}{(1-\rho)\theta}+1\right)^{-(1-\rho)/\rho}\left\{\int_S T_t(u)\left[mc_t(u)\varsigma(r,u)\right]^{-\theta} du\right\}^{-1/\theta}$$

8. Move on to the next period and repeat steps 1-7

Solution Algorithm - Solving for Utility Level, Lemma 3 (3/4)

- Solve for $\hat{u}_t$ from the following equation:

$$B_{1t}(r)\hat{u}_t(r)^{\frac{1}{\Omega}\left[\lambda\theta-\frac{\theta}{1+2\theta}\left[\alpha-1+\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\right]\theta\right]\right]+\frac{\theta(1+\theta)}{1+2\theta}}$$
$$=\kappa_1\int_S \hat{u}_t(s)^{\frac{1}{\Omega}\left[1-\lambda\theta+\frac{1+\theta}{1+2\theta}\left[\alpha-1+\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\right]\theta\right]\right]-\frac{\theta^2}{1+2\theta}}B_{2t}(s)\varsigma(r,s)^{-\theta}ds$$

where

$$B_{1t}(r)=\bar{a}(r)^{-\frac{\theta(1+\theta)}{1+2\theta}}\tau_t(r)^{-\frac{\theta}{1+2\theta}}H(r)^{\frac{\theta}{1+2\theta}\left[\alpha+\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\right]\theta\right]-\lambda\theta}$$
$$\times m_2(r)^{-\frac{1}{\Omega}\left[\lambda\theta-\frac{\theta}{1+2\theta}\left[\alpha-1+\left[\lambda+\frac{\gamma}{\xi}-[1-\mu]\theta\right]\right]\right]}$$
$$B_{2t}(r,s)=\bar{a}(s)^{\frac{\theta^2}{1+2\theta}}\tau_t(s)^{\frac{1+\theta}{1+2\theta}}H(s)^{\frac{\theta}{1+2\theta}-1+\lambda\theta-\frac{1+\theta}{1+2\theta}\left[\alpha-1+\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\right]\theta\right]}$$
$$\times m_2(s)^{-\frac{1}{\Omega}\left[1-\lambda\theta+\frac{1+\theta}{1+2\theta}\left[\alpha-1+\left[\lambda+\frac{\gamma_1}{\xi}-[1-\mu]\right]\theta\right]\right]}\varsigma(r,s)^{-\theta}$$

- Compute utility level:

$$u_t(r)=\hat{u}_t(r)/\left[\frac{\bar{L}}{\int_S m_2(v)^{-1/\Omega}\hat{u}_t(v)^{1/\Omega}dv}\right]^{\frac{F}{1-\frac{F}{\Omega}}}$$

where F is a constant (see page 976 of the paper)

Solution Algorithm - Backcasting (4/4)

- Backcasting: simulate the economy backward

1. Given τ_{t+1}, write τ_t as a function of $\bar{L}_t$ (equation (39))
2. Solve for $\bar{L}_t$ (equation (40))
3. Calculate τ_t and other endogenous variables in period t
4. Move on to period $t-1$ and repeat steps 1-3

Cruz and Rossi-Hansberg (2023)

Overview

- Application of Desmet, Nagy, and Rossi-Hansberg (2018) to climate change
- Three new components:

1. Endogenous law of motion for global population
2. Use of energy in production: fossil fuels generate CO_2 emissions, whereas clean sources do not
3. Carbon cycle: CO_2 emissions affect global and local temperature, which distort local fundamentals and natality rates

Natality

- $n_t(r)$ net off-springs for each household, which depends on local real income and temperature

$$n_t(r) = \eta(y_t(r), T_t(r))$$

- Law of motion for local population (before migration):

$$L'_{t+1}(r)H(r) = (1 + n_t(r))L_t(r)H(r)$$

- End-of-period population depends on natality and migration, which is shaped by activities across space

Energy

- Production requires energy, a CES bundle of fossil fuels and clean sources

$$q_t^\omega(r) = \phi_t^\omega(r)^{\gamma_1} z_t^\omega(r) \left(L_t^\omega(r)^\chi e_t^\omega(r)^{1-\chi}\right)^\mu, \quad e_t^\omega(r) = \left(\kappa e_t^{f,\omega}(r)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\kappa)\, e_t^{c,\omega}(r)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- Competitive local energy market such that price equals marginal cost $\mathcal{Q}$

$$\mathcal{Q}_t^f(r) = \frac{f(CumCO2_{t-1})}{\zeta_t^f(r)}, \quad \mathcal{Q}_t^c(r) = \frac{1}{\zeta_t^c(r)}$$

  - f increasing and convex: rising extraction cost
  - Cumulative emissions:

$$CumCO2_t = CumCO2_{t-1} + \int_S \int_0^1 e_t^{f,\omega}(v) H(v) d\omega dv$$

  - Rising efficiency ζ with global real GDP:

$$\frac{\zeta_t^j(r)}{\zeta_{t-1}^j(r)} = \left(\frac{y_t^w}{y_{t-1}^w}\right)^{v^j}$$

- Note that firms make static decisions on energy use

Carbon Cycle (1/2)

- Carbon emissions contribute to carbon stock:

$$S_{t+1} = S_{\text{pre-ind}} + \sum_{\ell=1}^{\infty} (1 - \delta_\ell) \left(E^f_{t+1-\ell} + E^x_{t+1-\ell} \right)$$

- Carbon stock affects global radiative forcing:

$$F_{t+1} = \varphi \log_2 \frac{S_{t+1}}{S_{\text{pre-ind}}} + F^x_{t+1}$$

- Global radiative forcing affects global temperature:

$$T_{t+1} = T_{\text{pre-ind}} + \sum_{\ell=0}^{\infty} \zeta_\ell F_{t+1-\ell}$$

- Global temperature affects local temperature (down-scaling):

$$T_t(r) - T_{t-1}(r) = g(r) \cdot (T_t - T_{t-1})$$

Carbon Cycle - Effects of Local Temperature (2/2)

- Local temperature affects...

  1. Natality rate (see above)

  2. Local fundamental amenity:

$$\bar{b}_t(r) = \left(1 + \underbrace{\Lambda^b(\Delta T_t(r), T_{t-1}(r))}_{\text{damage function}}\right) \bar{b}_{t-1}(r)$$

  3. Local fundamental productivity:

$$\bar{a}_t(r) = \left(1 + \underbrace{\Lambda^a(\Delta T_t(r), T_{t-1}(r))}_{\text{damage function}}\right) \left(\phi_{t-1}(r)^{\theta\gamma_1} \left[\int_S D(v,r)\bar{a}_{t-1}(v)dv\right]^{1-\gamma_2} \bar{a}_{t-1}(r)^{\gamma_2}\right)$$

- Agents react accordingly and choose energy use, closing the carbon cycle

Estimation - Energy Prices (1/4)

- Cost of fossil fuel extraction:

$$f(CumCO2_t) = \left(\frac{f_1}{f_2 + e^{-f_3(CumCO2_t - f_4)}}\right) + \left(\frac{f_5}{maxCumCO2 - CumCO2_t}\right)^3$$

  $maxCumCO2$ from IPCC (2013); f_1–f_5 estimated from matching Bauer et al. (2017)
- χ and κ estimated from firm FOCs:

$$\frac{w_0 \mathcal{Q}_0 E_0}{w_0 L_0} = \frac{\mu(1-\chi)}{\mu + \gamma_1/\xi}, \quad \left(\frac{\mathcal{Q}_0^f}{\mathcal{Q}_0^c}\right)\left(\frac{E_0^f}{E_0^c}\right)^{\frac{1}{\varepsilon}} = \frac{\kappa}{1-\kappa}$$

  $\varepsilon = 1.6$; other variables obtained from data/literature
- ζ estimated from firm FOCs again:

$$\zeta_0^f(r) = \left(\frac{\mu + \gamma_1/\xi}{\mu(1-\chi)\kappa}\right)\left(\frac{e_0(r)}{L_0(r)}\right)\left(\frac{e_0^f(r)}{e_0(r)}\right)^{\frac{1}{\varepsilon}} f(CumCO2_0)$$

$$\zeta_0^c(r) = \left(\frac{\mu + \gamma_1/\xi}{\mu(1-\chi)(1-\kappa)}\right)\left(\frac{e_0(r)}{L_0(r)}\right)\left(\frac{e_0^c(r)}{e_0(r)}\right)^{\frac{1}{\varepsilon}}$$

- v estimated from backcasting and fitting historical relative energy use

Estimation - Damage Functions (2/4)

- First, estimate $\bar{b}_t(r)$ and $\bar{a}_t(r)$ from inversion (as in Desmet, Nagy, and Rossi-Hansberg (2018))

- Identify (non-parametrically) the effect of temperature on local fundamentals:

$$\log(x_t(r)) = \sum_{j=1}^{J} \delta_j^x \cdot T_t(r) \cdot \underbrace{\mathbf{1}\{T_t(r) \geq \mathcal{T}_j\}}_{\text{temperature bin}} + \delta^z \cdot Z(r) \cdot \mathbf{1}\{x_t(r) = \bar{a}_t(r)/\phi_t(r)\}$$
$$+ \iota(g) \cdot \mathbf{1}\{x_t(r) = \bar{b}_t(r)\} + \iota_t(s_x) + \varepsilon_t(r)$$

Damage function: $\Lambda^x(\Delta T_t(r), T_{t-1}) = \delta^x(T_t(r))\Delta T_t(r)$

- Smooth non-parametric estimates using a logistic function

- Bliss-point: $\delta_j^x = 0$

Estimation - Natality (3/4)

- Let

$$n_t(r) = \eta^y(\log(y_t(r))) + \eta^T(T_t(r), \log(y_t^w))$$

where

$$\begin{aligned} \eta^y(\log(y_t(r))) &= \mathcal{B}(\log(y_t(r)); b^\ell) \cdot \mathbf{1}(\log(y_t(r)) < b^*) \\ &\quad + \mathcal{B}(\log(y_t(r)); b^h) \cdot \mathbf{1}(\log(y_t(r)) \geq b^*) \\ \eta^T(T_t(r), \log(y_t^w)) &= \frac{\mathcal{B}(T_t(r); b^T)}{1 + e^{b_w[\log(y_t^w) - \log(y_0^w)]}} \\ \mathcal{B}(\log(y_t(r)); b) &= b_0 + b_2 e^{-b_1(\log(y_t(r)) - b^*)^2} \end{aligned}$$

See footnote 36 for the shape of $\mathcal{B}(\cdot)$

- Parameters estimated from backcasting and fitting historical natality rates

Estimation - Carbon Cycle and Temperature Downscaling (4/4)

- Carbon cycle parameters estimated from matching the data
- Temperature downscaling scaler as a function of geographical attributes
- Parameters estimated from the correlation between global and local temperature

Solution Algorithm

- Solution algorithm similar to Desmet, Nagy, and Rossi-Hansberg (2018)
  - Aggregate dynamics with agents making static decisions
  - Solving the model by simulating forward (or backward)
- For problem set 2, okay to build on this model and add/drop elements
- What are the relevant counterfactuals to study energy transition?