Quantitative Spatial Models: Ahlfeldt, Redding, Sturm, and Wolf (2015)

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Plan for Today

- Quantification of Ahlfeldt, Redding, Sturm, and Wolf (2015)
- Equilibrium
- Estimation
- Solution algorithm for counterfactuals

Adjusting Location Characteristics

- Recall production and residential externalities

$$A_j = a_j \left(\sum_{s=1}^{S} e^{-\delta \tau_{js}} \left(\frac{H_{Ms}}{K_s} \right) \right)^{\lambda}, \quad B_i = b_i \left(\sum_{r=1}^{S} e^{-\rho \tau_{ir}} \left(\frac{H_{Rr}}{K_r} \right) \right)^{\eta}$$

- Cannot separate A_j from E_j, B_j from T_j

- Define adjusted location characteristics

$$\tilde{A}_j = A_j E_j^{\frac{\alpha}{\varepsilon}}, \quad \tilde{a}_j = a_j E_j^{\frac{\alpha}{\varepsilon}}$$

$$\tilde{B}_j = B_j T_j^{\frac{1}{\varepsilon}} \zeta_{Rj}^{1-\beta}, \quad \tilde{b}_j = b_j T_j^{\frac{1}{\varepsilon}} \zeta_{Rj}^{1-\beta}$$

$$\tilde{w}_j = w_j E_j^{\frac{1}{\varepsilon}}$$

$$\tilde{\varphi}_j = \tilde{\varphi}_j(\varphi_j, E_j^{\frac{1}{\varepsilon}}, \xi_j)$$

- Proceed with adjusted variables

General Equilibrium with Endogenous Location Characteristics (1/2)

- Exogenous parameters $\{\alpha, \beta, \mu, \varepsilon, \kappa, \lambda, \delta, \eta, \rho, \bar{U}, \tilde{a}_j, \tilde{b}_j, \tilde{\varphi}_j, K_j, \xi_j, \tau_{ij}\}$
- Endogenous variables $\{\pi_{Mj}, \pi_{Rj}, Q_j, q_j, \tilde{w}_j, \theta_j, H\}$

1. Free mobility

$$\bar{U} = \Gamma\left(\frac{\varepsilon - 1}{\varepsilon}\right)\left[\sum_{r=1}^{S}\sum_{s=1}^{S}\left(d_{rs}Q_r^{1-\beta}\right)^{-\varepsilon}\left(\tilde{B}_r\tilde{w}_s\right)^{\varepsilon}\right]^{\frac{1}{\varepsilon}}$$

2. Commuting flows, residential and workplace choice probabilities

$$\pi_{ij} = \frac{\left(d_{ij}Q_i^{1-\beta}\right)^{-\varepsilon}\left(\tilde{B}_i\tilde{w}_j\right)^{\varepsilon}}{\sum_{r=1}^{S}\sum_{s=1}^{S}\left(d_{rs}Q_r^{1-\beta}\right)^{-\varepsilon}\left(\tilde{B}_r\tilde{w}_s\right)^{\varepsilon}}, \quad \pi_{Ri} = \sum_{j=1}^{S}\pi_{ij}, \quad \pi_{Mj} = \sum_{i=1}^{S}\pi_{ij}$$

3. Commercial floor prices

$$q_j = (1-\alpha)\left(\frac{\alpha}{\tilde{w}_j}\right)^{\frac{\alpha}{1-\alpha}}\tilde{A}_j^{\frac{1}{1-\alpha}}$$

General Equilibrium with Endogenous Location Characteristics (2/2)

4. Commercial land market clearing

$$\left(\frac{(1-\alpha)\tilde{A}_j}{q_j}\right)^{\frac{1}{\alpha}} H_{Mj} = \theta_j L_j, \quad H_{Mj} = \pi_{Mj} H, \quad L_j = \tilde{\varphi}_j K_j^{1-\mu}$$

5. Residential land market clearing

$$(1-\beta)\left(\sum_{j=1}^{S} \frac{(\tilde{w}_j/d_{ij})^{\varepsilon}}{\sum_{s=1}^{S}(\tilde{w}_s/d_{is})^{\varepsilon}}\tilde{w}_j\right)\frac{H_{Ri}}{Q_i} = (1-\theta_i)L_i, \quad H_{Ri} = \pi_{Ri} H$$

6. No-arbitrage between alternative land uses

$$\begin{cases} \theta_i = 1 & \text{if } q_i > \xi_i Q_i \\ \theta_i \in [0,1] & \text{if } q_i = \xi_i Q_i \\ \theta_i = 0 & \text{if } q_i < \xi_i Q_i \end{cases}$$

Estimation Overview

- Data: H_{Mj}, H_{Ri}, $\mathbb{Q}_j$, K_j, τ_{ij} (notice that wages at the location level are not observed)

- Three sets of parameters to be determined

1. Calibrated parameters: $\{\alpha, \beta, \mu\}$
2. Parameters estimated from GMM: $\{\nu = \varepsilon\kappa, \varepsilon, \lambda, \delta, \eta, \rho\}$
3. Variables backed out from inversion: $\{\tilde{a}_j, \tilde{b}_j, \tilde{\varphi}_j\}$

Estimation - GMM (1/5)

- Calibrate $\alpha = 0.8$, $\beta = 0.75$, $\mu = 0.75$

- Adjusted production and residential fundamentals as functions of parameters

$$\Delta \log \left(\frac{\tilde{a}_{it}}{\bar{\tilde{a}}_t} \right) = (1-\alpha) \Delta \log \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t} \right) + \alpha \Delta \log \left(\frac{\tilde{w}_{it}(\nu)}{\bar{\tilde{w}}_t(\nu)} \right) - \lambda \Delta \log \left(\frac{\Upsilon_{it}(\delta)}{\bar{\Upsilon}_t(\delta)} \right)$$

$$\Delta \log \left(\frac{\tilde{b}_{it}}{\bar{\tilde{b}}_t} \right) = \frac{1}{\varepsilon} \Delta \log \left(\frac{H_{Rit}}{\bar{H}_{Rt}} \right) + (1-\beta) \Delta \log \left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}_t} \right) - \frac{1}{\varepsilon} \Delta \log \left(\frac{W_{it}(\nu)}{\bar{W}_t(\nu)} \right) - \eta \Delta \log \left(\frac{\Omega_{it}(\rho)}{\bar{\Omega}_t(\rho)} \right)$$

- Moment conditions
  1. Exogeneity of adjusted production and residential fundamentals

$$\mathbb{E} \left[\mathbb{I}_k \Delta \log \left(\frac{\tilde{a}_{it}}{\bar{\tilde{a}}_t} \right) \right] = 0, \quad \mathbb{E} \left[\mathbb{I}_k \Delta \log \left(\frac{\tilde{b}_{it}}{\bar{\tilde{b}}_t} \right) \right] = 0$$

  2. Matching commuting flows (identifies ν)
  3. Matching variance in log wages (identifies ε)

- Collect moments in the vector $\mathbf{m}$, parameters in the vector $\boldsymbol{\Lambda}$, data in the vector $\mathbf{X}$

- GMM estimator solves

$$\hat{\boldsymbol{\Lambda}}_{GMM} = \text{argmin}_{\boldsymbol{\Lambda}} \left(\frac{1}{N} \sum_{i=1}^{N} \mathbf{m}(\mathbf{X}_i, \boldsymbol{\Lambda})' \right) \mathbb{W} \left(\frac{1}{N} \sum_{i=1}^{N} \mathbf{m}(\mathbf{X}_i, \boldsymbol{\Lambda}) \right)$$

- Outer problem: search over the parameter space for $\boldsymbol{\Lambda}$ using an optimization routine

- Inner problem: given $\boldsymbol{\Lambda}$, solve for $\tilde{w}$ using a fixed-point algorithm

$$H_{Mjt} = \sum_{i=1}^{S} \frac{e^{-\nu \tau_{ijt}} \tilde{w}_{jt}^{\varepsilon}}{\sum_{s=1}^{S} e^{-\nu \tau_{ist}} \tilde{w}_{st}^{\varepsilon}} H_{Rit}$$

Estimation - GMM (3/5)

- Choice of weighting matrix $\mathbb{W}$
  - Identity matrix (consistent, inefficient)
  - Optimal weighting matrix (efficient)

$$\mathbb{W} = \mathbb{V}^{-1}, \quad \mathbb{V} = \mathbb{E}\left[\mathbf{m}(\mathbf{X}_i, \mathbf{\Lambda})\mathbf{m}(\mathbf{X}_i, \mathbf{\Lambda})'\right]$$

- Two-step GMM estimator
  - Step 1: GMM with $\mathbb{W} = \mathbb{I} \implies \hat{\mathbf{\Lambda}}$
  - Step 2: GMM with $\mathbb{W} = \hat{\mathbb{V}}^{-1} = \frac{1}{N}\sum_{i=1}^{N} \mathbf{m}(\mathbf{X}_i, \hat{\mathbf{\Lambda}})\mathbf{m}(\mathbf{X}_i, \hat{\mathbf{\Lambda}})' \implies \hat{\mathbf{\Lambda}}^{GMM}$

- Identification checks
  - Plot GMM objective as a function of parameters (convex?)
  - Plus structural residual as a function of distance grid cells (flat?)

Estimation - Inversion (4/5)

- It remains to estimate location fundamentals

1. Solve for $\tilde{w}_j$ using the fixed-point algorithm (inner problem)
2. Estimate $\tilde{A}_j$ and $\tilde{B}_j$

$$\tilde{A}_j = \left(\frac{1-\alpha}{\alpha}\tilde{w}_j\mathbb{Q}_j\right)^{1-\alpha}, \quad \tilde{B}_j = H_{Rj}^{\frac{1}{\varepsilon}}\mathbb{Q}_j^{1-\beta}\left(\sum_{s=1}^{S} e^{-\nu\tau_{is}}\tilde{w}_s^{\varepsilon}\right)^{-\frac{1}{\varepsilon}}$$

3. Estimate $\tilde{a}_j$ and $\tilde{b}_j$

$$\tilde{a}_j = \tilde{A}_j\left(\sum_{s=1}^{S} e^{-\delta\tau_{js}}\left(\frac{H_{Ms}}{K_s}\right)\right)^{-\lambda}, \quad \tilde{b}_j = \tilde{B}_j\left(\sum_{r=1}^{S} e^{-\rho\tau_{jr}}\left(\frac{H_{Rr}}{K_r}\right)\right)^{-\eta}$$

Estimation - Inversion (5/5)

4. Estimate $\tilde{\varphi}_i$ (adjusted density)

$$\tilde{L}_{Mi} + \tilde{L}_{Ri} = \tilde{\varphi}_i K_i^{1-\mu}$$

where

$$\tilde{L}_{Mi} = \left(\frac{\tilde{w}_i}{\alpha \tilde{A}_i}\right)^{\frac{1}{1-\alpha}} H_{Mi}, \quad \tilde{L}_{Ri} = (1-\beta)\left(\sum_{j=1}^{S} \frac{(\tilde{w}_j/d_{ij})^{\varepsilon}}{\sum_{s=1}^{S}(\tilde{w}_s/d_{is})^{\varepsilon}} \tilde{w}_j\right) \frac{H_{Ri}}{\mathbb{Q}_i}$$

Can back out θ_j (not observed) from $\tilde{L}_{Mi}$ and $\tilde{L}_{Ri}$

- Not so different from Owens, Rossi-Hansberg, and Sarte (2020)
- Key: identifying parameters in endogenous productivity and amenities using plausibly exogenous variations

Solution Algorithm for Counterfactuals (1/3)

- Fixed-point algorithm

1. Guess $\{\tilde{w}_j^0, \mathbb{Q}_j^0, \theta_j^0\}$
2. Solve for $\{\tilde{A}_j, \tilde{B}_j\}$ that is consistent with $\{\tilde{w}_j^0, \mathbb{Q}_j^0, \theta_j^0\}$
   2.1 Guess $\{\tilde{A}_j^0, \tilde{B}_j^0\}$
   2.2 Calculate commuting flows

$$\pi_{ij} = \frac{\left(d_{ij}(Q_i^0)^{1-\beta}\right)^{-\varepsilon}\left(\tilde{B}_i^0 \tilde{w}_j^0\right)^{\varepsilon}}{\sum_{r=1}^{S}\sum_{s=1}^{S}\left(d_{rs}(Q_r^0)^{1-\beta}\right)^{-\varepsilon}\left(\tilde{B}_r^0 \tilde{w}_s^0\right)^{\varepsilon}}$$

   2.3 Calculate employment and residence

$$H_{Mj} = \sum_{i=1}^{S} \pi_{ij} H, \quad H_{Ri} = \sum_{j=1}^{S} \pi_{ij} H$$

   2.4 Calculate implied $\{\tilde{A}_j^1, \tilde{B}_j^1\}$

$$A_j^1 = a_j \left(\sum_{s=1}^{S} e^{-\delta \tau_{js}} \left(\frac{H_{Ms}}{K_s}\right)\right)^{\lambda}, \quad B_i^1 = b_i \left(\sum_{r=1}^{S} e^{-\rho \tau_{ir}} \left(\frac{H_{Rr}}{K_r}\right)\right)^{\eta}$$

   2.5 Iterate steps 2.1-2.4 until convergence; get $\{\tilde{A}_j, \tilde{B}_j\}$ and $\{\pi_{ij}, H_{Mj}, H_{Rj}\}$

Solution Algorithm for Counterfactuals (2/3)

3. Calculate expected wage

$$\bar{\tilde{v}}_i = \sum_{j=1}^{S} \pi_{ij|i} \tilde{w}_j^0$$

4. Calculate output

$$Y_j = \tilde{A}_j^0 (H_{Mj})^{\alpha} (\tilde{\theta}_j^0 \tilde{\varphi}_j K_j^{1-\mu})^{1-\alpha}$$

5. Update wage

$$\tilde{w}_j^1 = \frac{\alpha Y_j}{H_{Mj}}$$

Solution Algorithm for Counterfactuals (3/3)

6. Update floor price

$$\mathbb{Q}_j^1 = \begin{cases} \dfrac{(1-\alpha)Y_j}{\tilde{\theta}_j^0 \tilde{\varphi}_j K_j^{1-\mu}}, & \text{if } \tilde{A}_j > 0 \\ \dfrac{(1-\beta)\bar{\tilde{v}}_j H_{Rj}}{(1-\tilde{\theta}_j^0)\tilde{\varphi}_j K_j^{1-\mu}}, & \text{if } \tilde{B}_j > 0 \end{cases}$$

7. Update allocation of floor space

$$\theta_j^1 = \begin{cases} 1, & \text{if } \tilde{A}_j > 0, \tilde{B}_j = 0 \\ 0, & \text{if } \tilde{A}_j = 0, \tilde{B}_j > 0 \\ \dfrac{(1-\alpha)Y_j}{\mathbb{Q}_j^1 \tilde{\varphi}_j K_j^{1-\mu}}, & \text{if } \tilde{A}_j > 0, \tilde{B}_j > 0 \end{cases}$$

8. Iterate steps 1-7 until convergence

- Possible multiple equilibria, similar to Owens, Rossi-Hansberg, Sarte (2020)
- Initialize with baseline $\Longrightarrow$ the closest equilibrium to the baseline